Magnetotransport in an array of magnetic antidots

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Abstract. Classical transport properties of an electron, moving in plain, in an array of magnetic antidots has been calculated. Homogeneous magnetic field $\vec{B} = B\vec{e}_z$ fills the whole space except of cylinders of radius r_0, which are arranged in a square lattice with a lattice constant a. The magnetoresistance shows additional peaks and minima by varying the strength of the magnetic field, according to pinned orbits at antidots and to propagating orbits in transport direction, respectively.

The dynamics of charged particles in spatially modulated magnetic field give rise to a variety of interesting phenomena. The motion of ballistic electrons in modulated magnetic field is also believed to be closely related to the motion of composite fermions in a density modulated 2DEG in the fractional quantum Hall regime [1, 2]. Commensurability effects in weak periodic magnetic fields were predicted [3, 4] and observed for modulation in one [5] and two directions [7] experimentally. Such periodic magnetic field can be generated by using either patterned superconducting [8] or ferromagnetic gates [5],[6]. Closely related to our system are antidot arrays, which consist of periodically arranged voids in an electron gas. Charged particles move in this two dimensional potential landscape under the influence of perpendicular magnetic field and are scattered on the antidots. [9] Additional peaks in the low field magnetoresistance have been observed [10] in these antidot arrays and explained [11] by a pinning mechanism of classical circular orbits enclosing 1,2,4,9 and 21 antidots in terms of nonlinear dynamics. The nature of magnetoresistance peaks are caused by islands of regular motion due to nonlinear resonances.

In the present paper we study the magnetotransport in an array of *magnetic antidots*. Instead of antidot scatterers we introduce circular regions into the system with zero magnetic field inside. Inside the magnetic antidots the electron moves in a field-free region and outside is its motion described by the Lorentz force. As demostrated in Fig. 1 the inhomogenity of the z-componetnt of the magnetic field for one magnetic antidot is given by

$$B(r)\vec{e}_z = \begin{cases} -B\vec{e}_z & \text{for} \quad r > r_0 \\ 0 & \text{else} \end{cases}$$

and these antidots are arranged in a rectangular array of lattice constant a. In the recent experiments on modulated magnetic fields, the period of modulation lies by 700 nm and is larger then the Fermi wave length, but smaller then the elastic mean free path (10 μm) and thus the dynamics of the wave packet approaches the classical limit. Magnetic antidot array could be realized using supeconducting materials, which would be maped on two dimensional electron gas in a rectangular array arrangement. By applying magnetic field, in circular region of superconducting material, the applied magnetic field would be swept out and the electrons would move in a system described in this paper. In the calculation we will use a system, that would be produced by an ideal vortex with a constant magnetic field outside and zero magnetic field inside, instead of more realistic exponential distribution of the magnetic. We consider also one elctron approximation.

The classical trajectory is found as the solution to Newtons' equation of motion with the force given by the Lorentz expression $\vec{F} = e\vec{v} \times \vec{B}$ for a particle of charge e. It consist, as well known of straight line segments inside the antidot and an arc of a circle outside, with the radius of the curvature given by the cyclotron radius $r_c = \frac{v}{\omega_c}$ with v being the particle velocity and $\omega_c = \frac{eB}{m}$ the cyclotron frequency. There exist also undisturbed circular orbits, which do not intersect any antidot. The classical approximation for the dynamics of an electron wave packet in a modulated magnetic field is described by the

Hamiltonian for one magnetic antidot

$$H(x,y,p_x,p_y) = \begin{cases} [(p_x + eBy/2)^2 + (p_y - eBx/2)^2]/2m & \text{for} \quad r > r_0 \\ \vec{p}^2/2m & \text{else} \end{cases}$$

where m is the effective mass of the electron. First we introduce some characteristic parameters of the system and express all quantities in dimensionless units: the coordinate $x\prime = x/a$, $y\prime = y/a, t\prime = t/\tau_0$ and $H\prime = H/\epsilon_F$ with ϵ_F, the Fermi energy, $\tau_0 = (\epsilon_F/ma^2)^{1/2}$. The magnetic field is scaled by B_0 at which the cyclotron radius corresponds to $a/2$. Calculating the Poisson brackets and omitting the primes we obtain following equations of motion for one magnetic antidot

$$\dot{x} = v_x, \quad \dot{y} = v_y$$

$$\dot{v}_x = \begin{cases} B/B_0 v_y & \text{for} \quad r > r_0 \\ 0 & \text{else} \end{cases}$$

$$\dot{v}_y = \begin{cases} -B/B_0 v_x & \text{for} \quad r > r_0 \\ 0 & \text{else} \end{cases}$$

Choosing $r_0 = 0.3a$ and using a numerical integration method it is possible to calculate the trajectories of the electron for different values of magnetic field. We investigate the motion in phase space (x, y, v_x, v_y) by means of Poincaré surface of section at ($x = 0$ mod a). Since the energy $E = v_x^2/2m + v_y^2/2m$ of the system is conserved, we have to consider the surface of section (y, v_y) for $v_x > 0$ and $v_x < 0$ separately. The initial conditions (y_i, v_{yi}) lead for $v_x > 0$ and $v_x < 0$ to different trajectories and the corresponding two surfaces of section have mirror symmetry. Fig.2 shows the surface of section for $v_x > 0$ for three different values of magnetic field $B/B_0 = 0.5, 0.6$ and 1.0, the corresponding cyclotron radius is $r_c = a,\ 0.83a,\ a/2$. The whole phase space consits at $B/B_0 = 0.5$ of chaotic sea as shown in Fig.2 (c), one of possible chaotic trajectories in coordinate space is shown in Fig.3 (IV). At $B/B_0 = 0.6$ we found periodic orbits, which intersect two magnetic antidots as shown in Fig.3 (III). These orbits occupy a relative small portion in the phase space as shown in Fig.2 (b), label(III). An other type of periodic orbits, which intersect four magnetic antidots is shown in Fig.3 (V). The fingerprint of this orbit in the surface of section is shown in Fig.2 (a), label (V). When the cyclotron radius approaches half of the lattice constant, a different type of periodic orbit becomes dominat. These orbits are pinned around one magnetic antidot as shown in Fig.3 trajectory (II) and build in the surface of section a family of eliptic fixed points in a circular region at $(y, v_y) = (0.5, 0)$ of diameter r_0 as shown in Fig.2 (a), region (II). These orbits can also intersect one magnetic antidot for higher magnetic fileds as shown in Fig.3 (I) and dominate the dynamics of the system. The orbits of type (I),(II) and (V) are present in a wide sector of magnetic field strength $B/B_0 \in]0.8, 1.7]$. For lower values of magnetic field $B/B_0 = 0.3$ periodic orbits, which intersect eight magnetic antidots occupy a relative small region (10%) in phase space. For $B/B_0 > 2.3$ all possible orbits are pinned, and no classical chaotic trajectories exist anymore. The inset of Fig.4 (a)

shows the portion p of regular orbits in dependence of magnetic field.
At low temperatures elastic impurity scatterings with mean scattering time τ have to be considered. Using the classical Kubo formula [11] which includes these scattering, we can calculate the conductivity σ_{ij} in dependence of magnetic field.

$$\sigma_{ij}(0,\tau) = \frac{ne^2}{k_B T}\int_0^\infty dt e^{-1/\tau} < v_i(t)v_j(0) >_{\Gamma_c}$$

$< v_i(t)v_j(0) >$ is the velocity correlation function averaged over chaotic part of the phase space Γ_c, k_B is the Boltzmann constant, T temperature and n the 2D electron density. The magnetoresistance can be obtain by inversion of the conductivity tensor $R_{xx} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ and $R_{xy} = \sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$. We have calculated the magnetoresistance R_{xx} and the Hall resistance R_{xy} in dependence of magnetic field for a typical value of $\tau/\tau_0 = 30$. The magnetoresistance shown in Fig.4 (a) displays a dominant peak at $B/B_0 \approx 1$ which corresponds to pinned orbits around one antidot. According to the portion of pinned orbits, see the inset of Fig.4 (a) we conclude, that this peak is caused by these periodic orbits. We should expect also a peak at $B/B_0 \approx 0.3$ and 0.6 as in the case of antidot lattices, but instead we observe a minimum at $B/B_0 = 0.8$. The main difference to antidot lattices, where electrons are scattered on antidots, is that the sign of the angular momentum of an electron, moving in a magnetic antidot array remains conserved. The magnetoresistance is not only caused by islands of regular motion in the phase space (pinned orbits), but also by orbits, which propagate in the direction of transport. For $B/B_0 = 0.8$ we found orbits, which propagate in transport direction, as shows in Fig. 3 (b) and explain the minimun in the magnetoresistance at this strength of magnetic field. We obtain even $R_{xx}/R_{xx}(0) < 1$, the resistance lies lower then, for the case of zero magnetic field. The Hall resistance as shown in Fig. 4 (b) shows steps at magnetic field values for which the magnetoresistance R_{xx} lies in minimum.
In conclusion, classical transport properties of electrons, moving in a magnetic antidot array has been explained by pinned orbits at antidots and by propagating orbits in transport direction.

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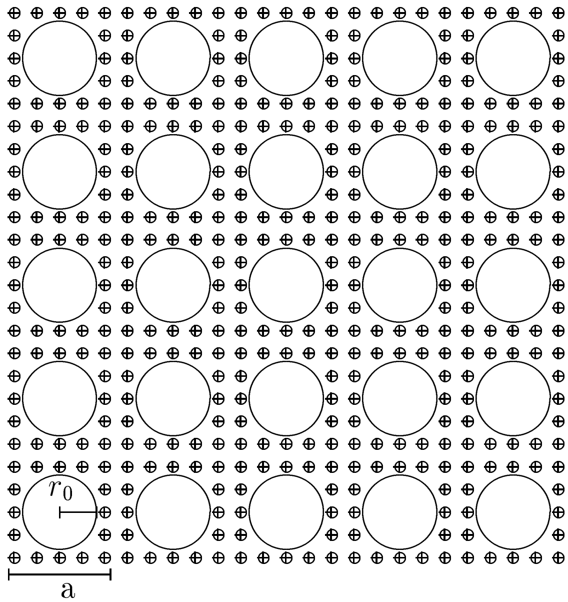
Figure captions

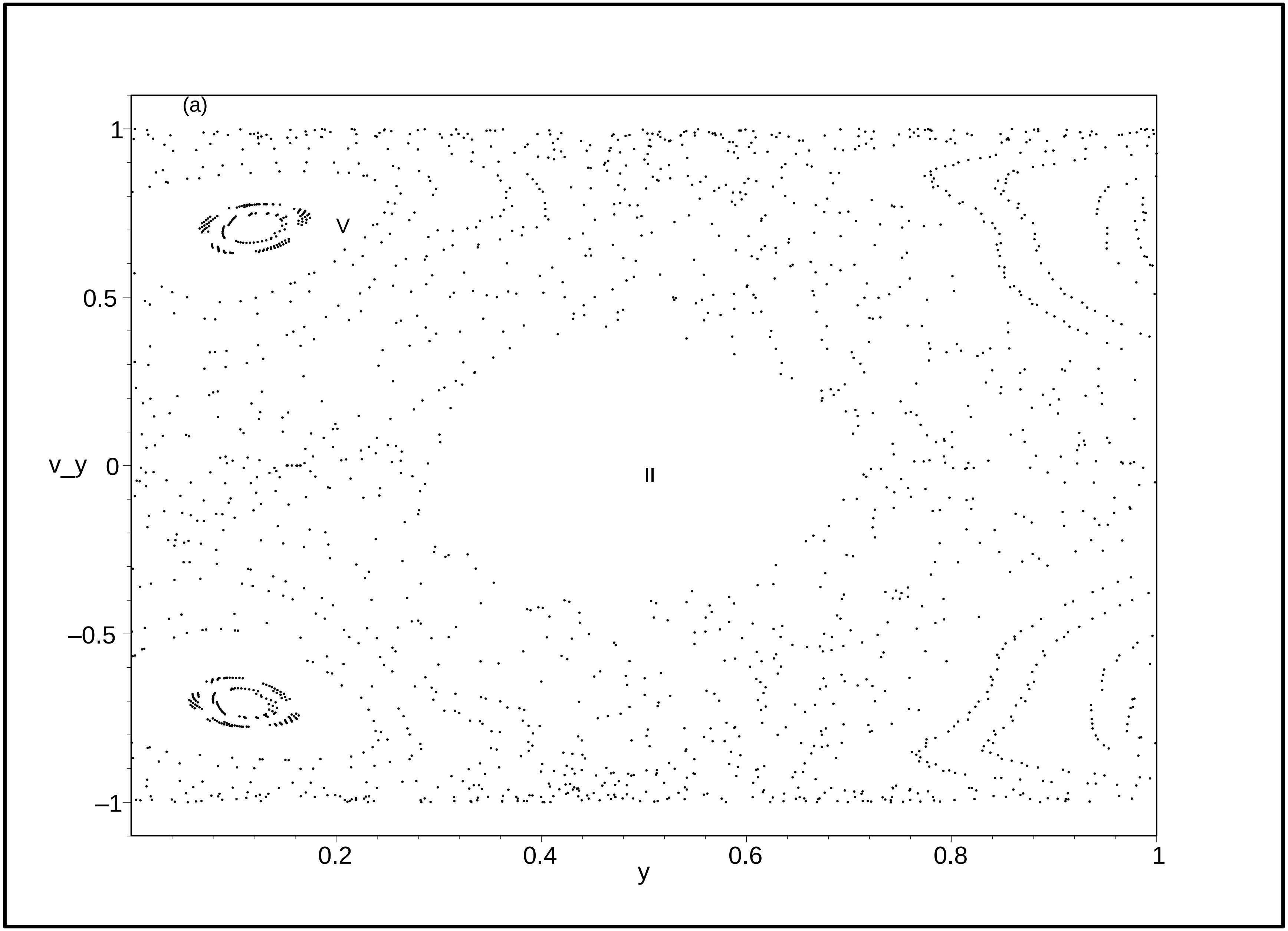
Figure 1. Schematic model of the magnetic antidot array. Inside the antidot is fieldfree region and outside acts homogeneous magnetic field in z-direction.The radius of the antidot is r_0 and the lattice constant is a.

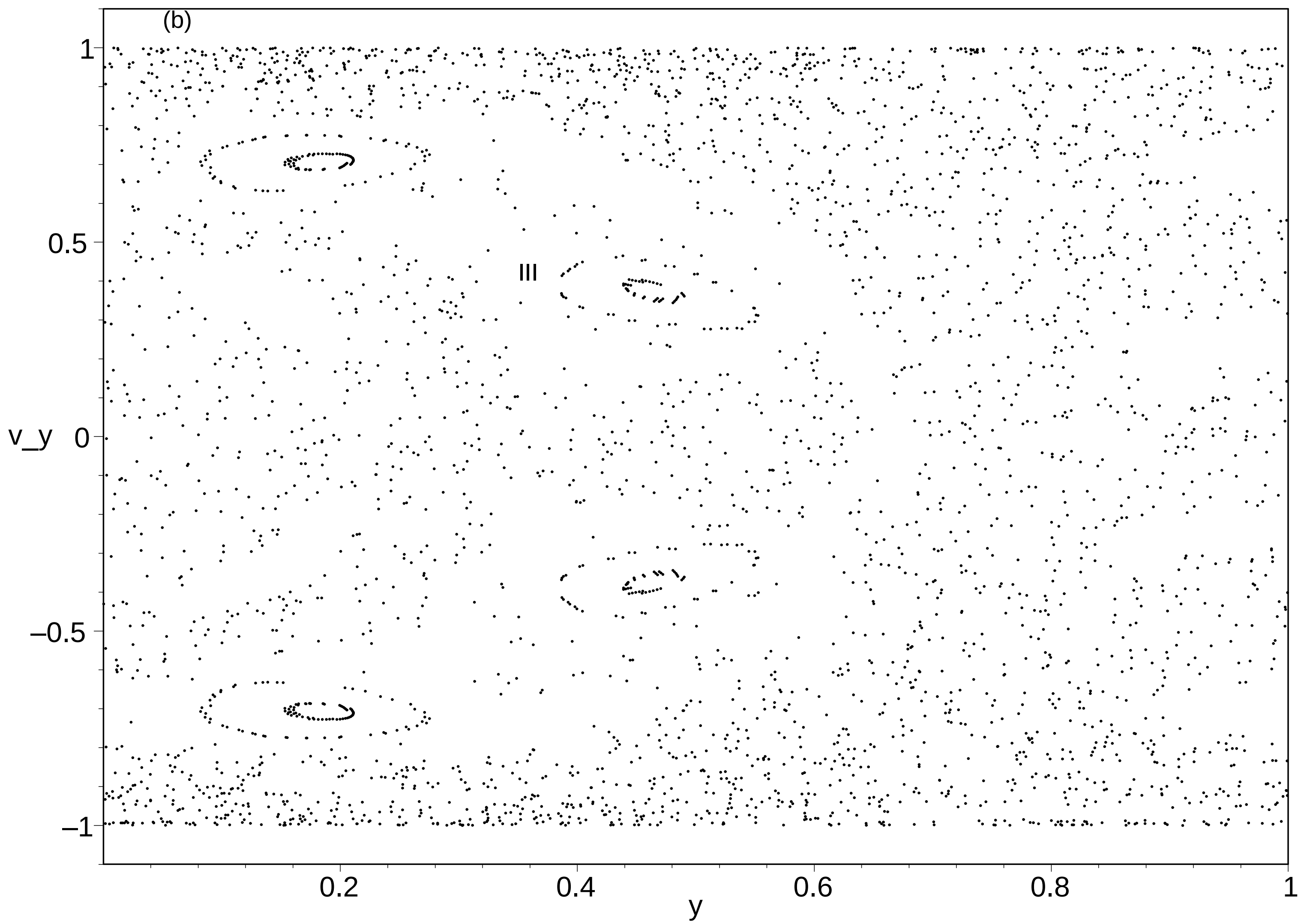
Figure 2. The Poincaré surface of section for three differnt values of magnetic field strength. (a) $B/B_0 = 1.0$, a circular region at $(y, v_y) = (0.5, 0)$ is filled by elliptic fix points, which correspond to pinned orbit of type II. Pinned orbits of type V are still present. (b) $B/B_0 = 0.6$, islands of regular motion corresponding to pinned orbits of type III and V are present. (c)$B/B_0 = 0.5$, the chaotic sea fills the whole phase space.

Figure 3. Different types of trajectories in magnetic antidot array. (a) Pinned orbits, which cause peaks in magnetoresistance. (b) Propagating orbits in transport direction, which cause minima in magnetoresistance.

Figure 4. The magnetoresistance (a) and Hall resistance (b) in dependence of the strength of the magnetic field. The minimum at $B/B_0 = 0.8$ in R_{xx} is caused by propagating orbits in transport direction and the the peak at $B/B_0 \approx 1.0$ is caused by pinned orbits around one magnetic antidot. The inset shows the amount of islands of regular motion in phase space. Steps in R_{xy} correspond to negative gradients in R_{xx}.


r_0
a


(a)
1
0.5
0
–0.5
–1
v_y
0.2
0.4
0.6
0.8
1
y
V
II


(b)
III
v_y
1
0.5
0
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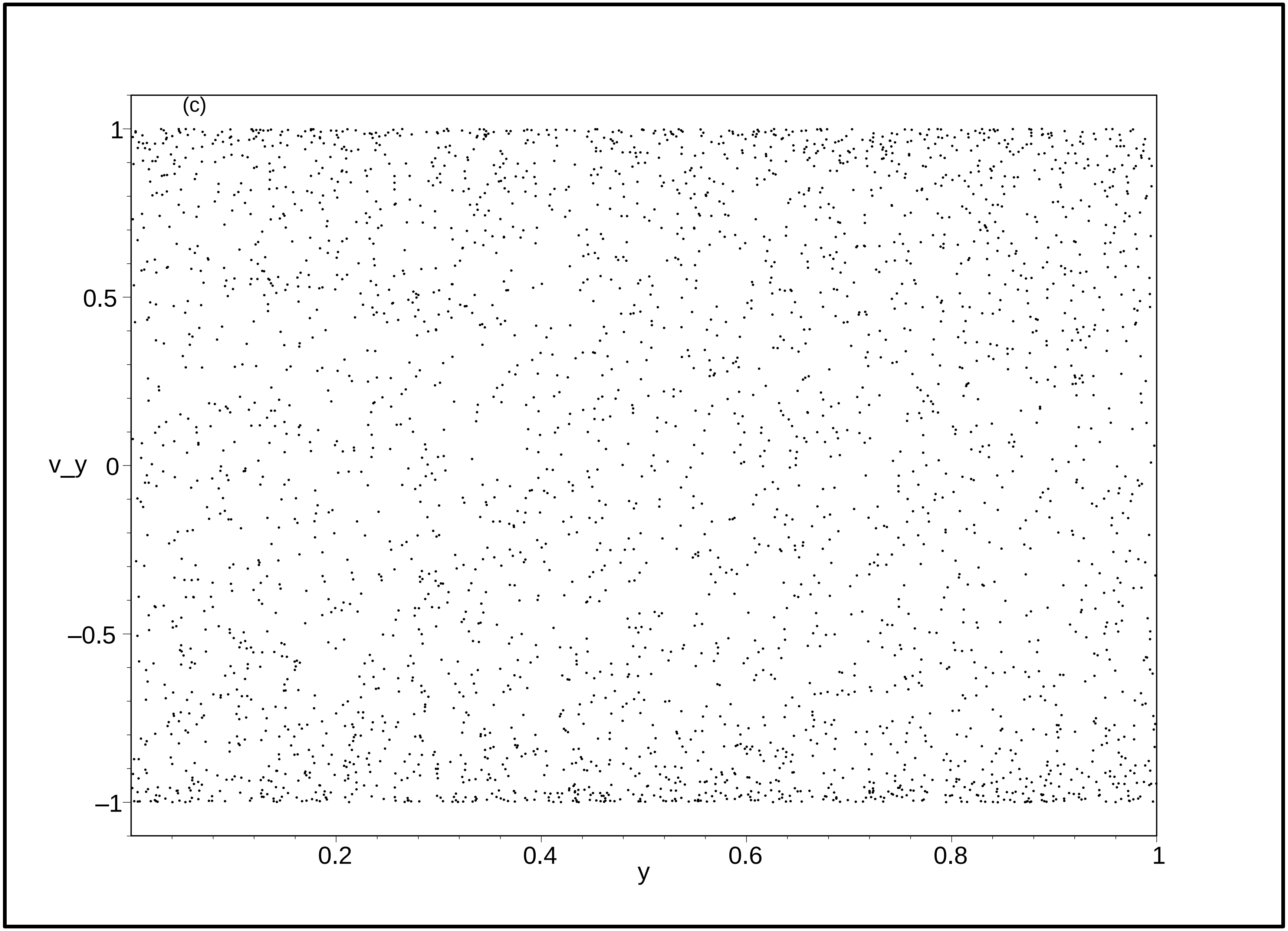
–1
0.2
0.4
0.6
0.8
1
y


(c)
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v_y
0
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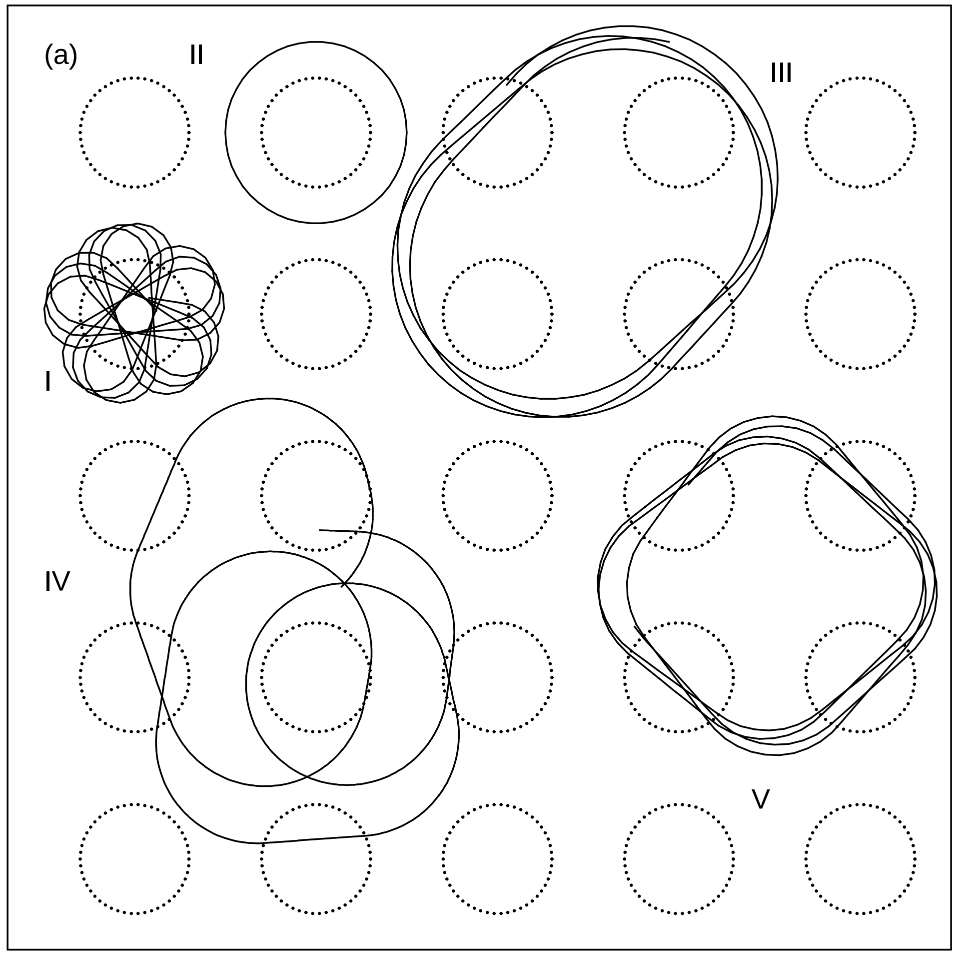
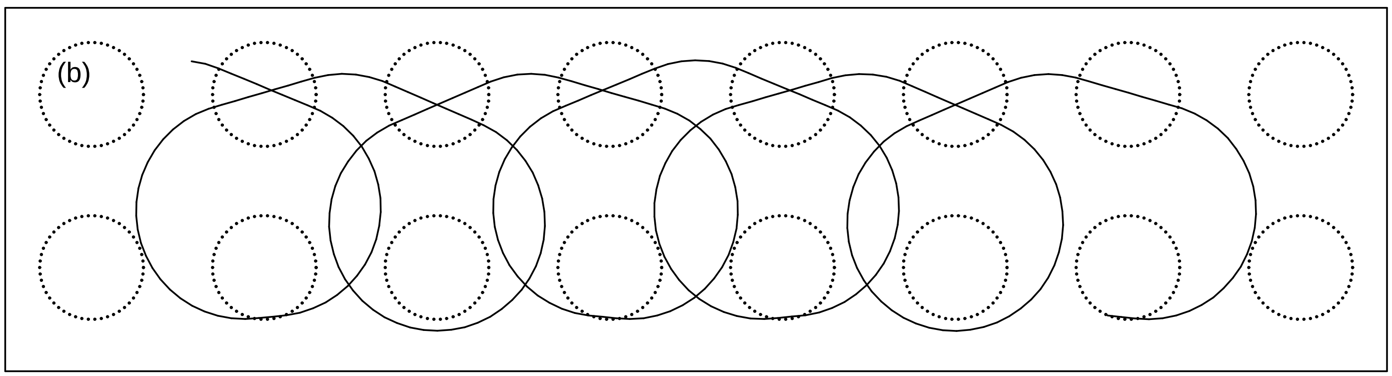
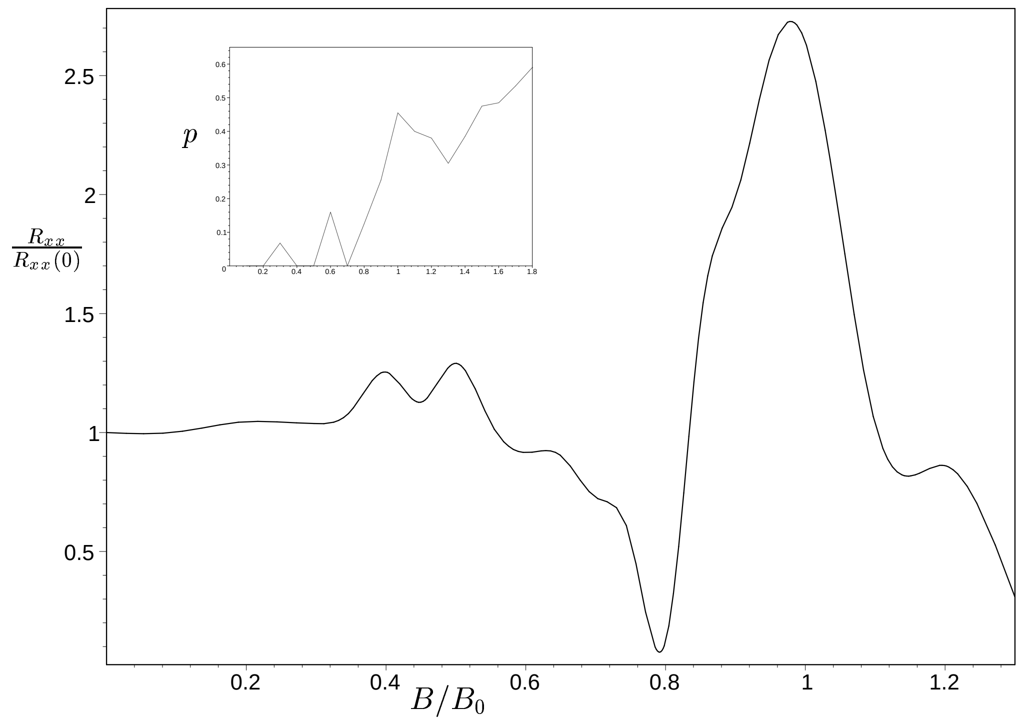
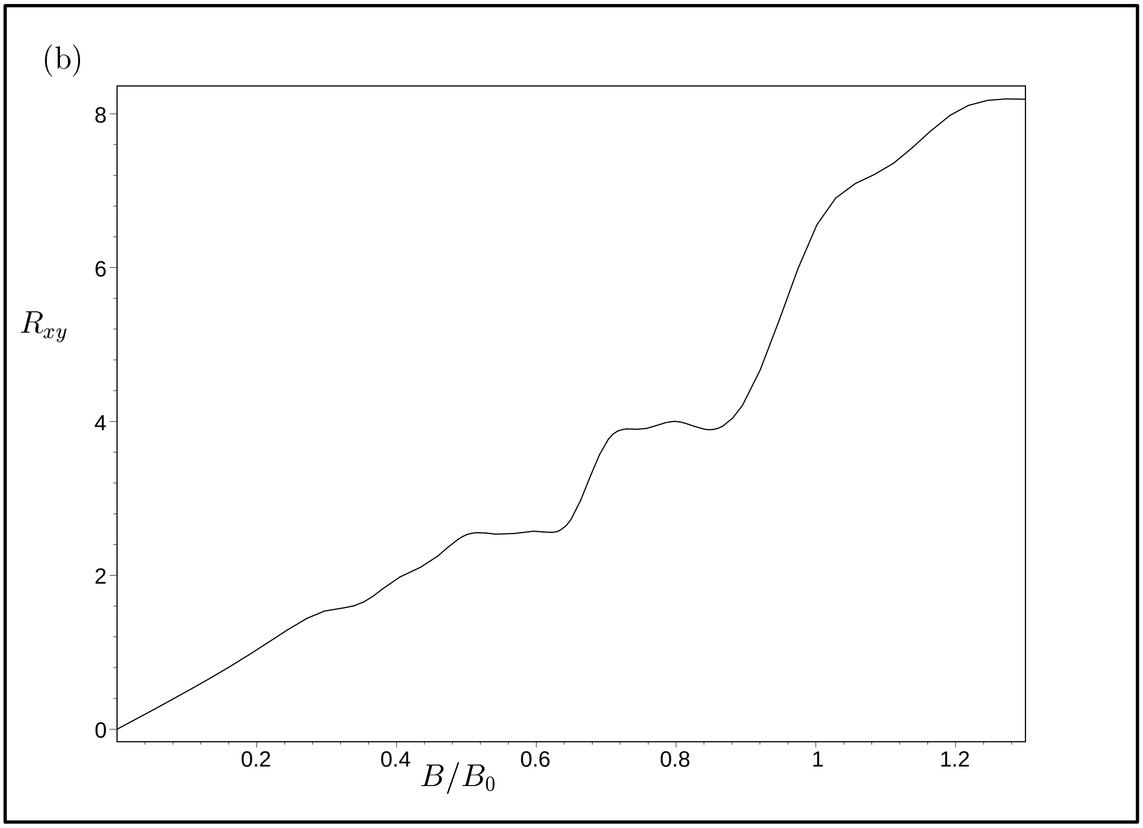
–1
0.2
0.4
0.6
0.8
1
y


(a)
II
III
I
IV
V


(b)

(a)





(b)
R_{xy}
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
B/B_0